12. Let $f'(a) = \lim_{h \to 0} \frac{\sqrt{4+h}-2}{h}$ $\rightarrow f(c)$

a. What is $f(x)$? $\sqrt{x}$

b. What is the value of $a$? $4$

c. Evaluate the given limit. $\frac{1}{4}$

$f(c+h)$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

3. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

A) $e^2$ B) 1 C) 1/2 D) 0 E) nonexistent

$f(c+h)$

$f(c)$

$$\lim_{h \to 0} \frac{\ln(2+h) - \ln 2}{h}$$

$c = 2$ $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

8. $\sec y = x$

$y = \sec^{-1} x$

$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$

3. $\lim_{A\to 0} \frac{1-\cos A}{2\sin^2 A}$ is

a) 0 b) 1/8 c) 1/4 d) 1 e) nonexistent

$$\lim_{A\to 0} \frac{1-\cos A}{2\sin^2 A} = \lim_{A\to 0} \frac{1-\cos A}{2(1-\cos^2 A)}$$

$$= \lim_{A\to 0} \frac{\cancel{1-\cos A}}{2\cancel{(1-\cos A)}(1+\cos A)} = \frac{1}{4}$$

$\cos A \to 1$

Part III – Free response (10 pts each)

9. Let $y = \ln\left((x-1)^3\sqrt{x}\right)$

$$y = \ln(x-1)^3 + \ln\sqrt{x}$$

$$y = 3\ln(x-1) + \frac{1}{2}\ln x$$

$$y' = \frac{3}{x-1} + \frac{1}{2x}$$

$$y = \ln\left(\frac{x^2-4}{2x+5}\right)$$

$$y = \ln(x^2-4) - \ln(2x+5)$$

$$y' = \frac{2x}{x^2-4} - \frac{2}{2x+5}$$

$$y = \sin^{-1}(\cos x)$$

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$y' = \frac{1}{\sqrt{1-\cos^2 x}} \cdot (-\sin x)$$

$$= \frac{-\sin x}{\sqrt{\sin^2 x}} = \frac{-\sin x}{|\sin x|} = \pm 1$$

$\sin^{-1}\frac{1}{2} = x = \frac{\pi}{6}$

$\sin x = \frac{1}{2}$

b. $y = x\tan^{-1}(3x)$ at $x = \frac{1}{3}$

$$y' = \tan^{-1}(3x) + x\left(\frac{1}{1+9x^2}\cdot 3\right)$$

$$= \tan^{-1}(3x) + \frac{3x}{1+9x^2}\Bigg|_{x=\frac{1}{3}}$$

$$= \tan^{-1}(1) + \frac{1}{1+9\left(\frac{1}{3}\right)^2}$$

$$= \frac{\pi}{4} + \frac{1}{2}$$

12. Find the derivative at the given point.

a. $\cos(xy) = x + y$ at $(1, 0)$

$$-\sin(xy)\left(y + x\frac{dy}{dx}\right) = 1 + \frac{dy}{dx} \qquad \frac{dy}{dx} = -1$$

$$\underbrace{-\sin(0)}_{0}\cancel{\left(\quad\right)} = 1 + \frac{dy}{dx}$$

11. Find the points on the ellipse $x^2 + 4y^2 = 1$ where the tangent line has slope 1.

$y'$

$2x + 8yy' = 0$

$2x + 8y(1) = 0$

$x = -4y$

$(-4y)^2 + 4y^2 = 1$

$16y^2 + 4y^2 = 1$

$20y^2 = 1$

$y = \pm\sqrt{\frac{1}{20}}$

$\sqrt{\frac{1}{20}} = \frac{1}{\sqrt{20}}$

$= \frac{1}{2\sqrt{5}}$

$= \frac{\sqrt{5}}{10}$

$y = \sqrt{\frac{1}{20}} \rightarrow x = -4\sqrt{\frac{1}{20}}$

$y = -\sqrt{\frac{1}{20}} \rightarrow x = 4\sqrt{\frac{1}{20}}$

$\left(-4\sqrt{\frac{1}{20}}, \sqrt{\frac{1}{20}}\right)$

$\left(4\sqrt{\frac{1}{20}}, -\sqrt{\frac{1}{20}}\right)$

4. If $xy^2 + 2xy = 8$, then, at the point (1, 2), $y'$ is

A) $-\frac{5}{2}$ B) $-\frac{4}{3}$ C) $-1$ D) $-\frac{1}{2}$ E) 0

$$y^2 + 2xyy' + 2y + 2xy' = 0$$

$$4 + 4y' + 4 + 2y' = 0$$

$$6y' = -8$$

$$y' = -\frac{8}{6} = -\frac{4}{3}$$

1. If $y = \ln((x+1)^2 + y^2)$, then the value of $\frac{dy}{dx}$ at the point $(0, 0)$ is

A) 0 B) ½ C) 1 D) 2 E) undefined

$$y' = \frac{1}{(x+1)^2 + y^2}\left(2(x+1) + 2yy'\right)$$

$$= \frac{1}{1}(2+0) = 2$$