

limits w/ trigs.

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$* \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin 2x}{3x} = \frac{\sqrt{3}}{\pi}$$

$$1) \lim_{x \rightarrow \pi} \frac{\sin(x-\pi)}{x-\pi} = 1$$

$$\frac{\sin(2 \cdot \frac{\pi}{6})}{3(\frac{\pi}{6})} = \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{2}}$$

$$2) \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$$

$$3) \lim_{x \rightarrow 0} x \csc(x) = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \frac{a}{b}$$

$$\lim_{x \rightarrow 0} \frac{\sinh x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin x}{x(1 + \cos x)} = 0$$

Properties of \ln .

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^c = c \cdot \ln x$$

$$\ln e^k = k$$

$$e^{\ln k} = k$$

$$|x| = \sqrt{x^2}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} \approx \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6}}{x^3} = \lim_{x \rightarrow -\infty} \frac{3|x^3|}{x^3}$$

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10. For the given equation below,

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = f'(c)$$

- a) What is $f(x)$? e^x
 b) What is the value of c ? 0
 c) Evaluate the given limit expression. |

$$e^h = e^{0+h}$$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f'(0) = e^0 = 1$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} = f'(c)$$

$$\text{if } f(x) = e^x$$

$$f(0+h) = e^{0+h}$$

9. $\lim_{x \rightarrow \infty} \frac{\sin(10x)}{x-4} = 0$

Handwritten notes in green:

- A box around $\sin(10x)$ with an arrow pointing to $-1 \leq \sin \leq 1$.
- A circle around $x-4$ with an arrow pointing to ∞ .
- The result $= 0$ is double-underlined.

3. $\lim_{A \rightarrow 0} \frac{1 - \cos A}{2 \sin^2 A}$ is

a) 0

b) 1/8

c) 1/4

d) 1

e) nonexistent

$$\lim_{A \rightarrow 0} \frac{1 - \cos A}{2(1 - \cos^2 A)} = \lim_{A \rightarrow 0} \frac{\cancel{1 - \cos A}}{2(\cancel{1 - \cos A})(1 + \cos A)}$$

$$= \lim_{A \rightarrow 0} \frac{1}{2(1 + \cos A)} = \frac{1}{4}$$

$$a^2 - b^2 = (a - b)(a + b)$$