33. $y = \ln|\sec 5x + \tan 5x|$

$$y' = \frac{1}{\sec 5x + \tan 5x} \cdot \left((\sec 5x \tan 5x)5 + \sec^2 5x \cdot 5\right)$$

$$= \frac{5\sec 5x\left(\cancel{\tan 5x + \sec 5x}\right)}{\cancel{\sec 5x + \tan 5x}} = 5\sec 5x$$

$(\ln x)' = \frac{1}{x}$

$(\sec x)' = \sec x \tan x$

$(\tan x)' = \sec^2 x$

25. $\sin(xy) = x^2 - y$

$$\cos(xy)\left(1\cdot y + x\frac{dy}{dx}\right) = 2x - \frac{dy}{dx}$$

$$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 2x - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} = 2x - y\cos(xy)$$

$$\frac{dy}{dx}\left(x\cos(xy) + 1\right) = 2x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy) + 1}$$

53. Find y'' if $x^6 + y^6 = 1$.

$$6x^5 + 6y^5 y' = 0$$

$$y' = \frac{-6x^5}{6y^5} = \frac{-x^5}{y^5}$$

$$y'' = \frac{-5x^4y^5 + (+x^5)(5y^4)\,y'}{y^{10}}$$

$$= \frac{\left[-5x^4y^5 + x^5\, 5\cancel{y^4}\left(-\frac{x^5}{\cancel{y^5}\, y}\right)\right]y}{\left[y^{10}\right]y}$$

$$= \frac{-5x^4y^6 - 5x^{10}}{y^{11}}$$

66. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

$y' = 1$

$2x + 4y(y') = 0$

$2x + 4y = 0$

$2x = -4y$

$x = -2y$

$(-2y)^2 + 2y^2 = 1$

$4y^2 + 2y^2 = 1$

$6y^2 = 1$

$y^2 = \frac{1}{6}$

$y = \pm\sqrt{\frac{1}{6}} = \frac{\pm\sqrt{6}}{6}$

when $y = \frac{\sqrt{6}}{6}$, $x = -2\left(\frac{\sqrt{6}}{6}\right) = -\frac{\sqrt{6}}{3}$

$y = -\frac{\sqrt{6}}{6}$, $x = \frac{\sqrt{6}}{3}$

31. $y = x\tan^{-1}(4x)$

$$y' = 1\cdot\tan^{-1}(4x) + x\left(\frac{4}{1+16x^2}\right)$$

$$= \tan^{-1}(4x) + \frac{4x}{1+16x^2}$$

60–61 Find equations of the tangent line and normal line to the curve at the given point.

60. $x^2 + 4xy + y^2 = 13, \quad (2, 1)$

$2x + 4(y + xy') + 2yy' = 0$

$4 + 4(1 + 2y') + 2y' = 0$

$4 + 4 + 8y' + 2y' = 0$

$10y' = -8$

$y' = -\frac{4}{5}$

Eq. of T

$y - 1 = \frac{-4}{5}(x - 2)$

Eq. of N

$y - 1 = \frac{5}{4}(x - 2)$