

Let $\sin^{-1} x = y$ $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\cos^{-1} x = y$ $0 \leq y \leq \pi$

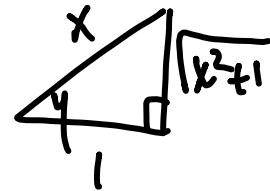
$\tan^{-1} x = y$ $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$$\text{Let } y = \sec^{-1} x$$

$$0 \leq y \leq \pi$$

Q1 & Q2

$$x = \sec y \rightarrow \cos y = \frac{1}{x}$$



$$x' = (\sec y)'$$

$$1 = \sec y \cdot \tan y \frac{dy}{dx}$$

$$\sec y = x$$

$$\tan y = \sqrt{x^2 + 1}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \cdot \tan y} = \frac{1}{|x| \sqrt{x^2 + 1}}$$

$$\text{if } y \in Q1$$

$$\sec y > 0$$

$$\tan y > 0$$

$$\sec y \cdot \tan y > 0$$

$$y \in Q2$$

$$\sec y < 0$$

$$\tan y < 0$$

$$\sec y \tan y > 0$$

DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx} (\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} (\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} (\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (\cot^{-1}x) = -\frac{1}{1+x^2}$$

45. $y = \tan^{-1}\sqrt{x}$

$$\begin{aligned}
 (\sqrt{x})' &= (x^{\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}} \\
 (\tan^{-1}x)' &= \frac{1}{1+x^2}
 \end{aligned}$$

$$y' = \frac{1}{1+(\sqrt{x})^2} \cdot \left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}$$

$$(\sqrt{x})^2 = x$$

$$= \frac{1}{(1+x)2(\sqrt{x})}$$

$$53. y = \cos^{-1}(e^{2x})$$

$$(\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}$$

$$y' = \frac{-1}{\sqrt{1-(e^{2x})^2}} (e^{2x} \cdot 2)$$

$$= \frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$$

$$49. G(x) = \sqrt{1-x^2} \arccos x$$

$$\left(\sqrt{1-x^2}\right)' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}}$$

$$\left(\cos^{-1} x\right)' = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dG}{dx} = \frac{-x}{\sqrt{1-x^2}} \cdot \cos^{-1} x + \frac{-1}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}$$

$$= \frac{-x \cos^{-1} x}{\sqrt{1-x^2}} - 1$$

$$\text{11. } F(t) = \ln \frac{(2t + 1)^3}{(3t - 1)^4}$$

$$= \ln (2t+1)^3 - \ln (3t-1)^4$$

$$= 3 \ln (2t+1) - 4 \ln (3t-1)$$

$$F' = 3 \left(\frac{1}{2t+1} \cdot 2 \right) - 4 \left(\frac{1}{3t-1} \cdot 3 \right)$$