Derivatives w/ Trig.

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

$\frac{x}{2} = \frac{1}{2}x$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ $\boxed{\text{(E) } -\frac{1}{2}\cos\left(\frac{x}{2}\right)}$

$$\frac{dy}{dx} = \cancel{2}\left(-\sin\left(\frac{x}{2}\right)\right)\cancel{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\cos\left(\frac{x}{2}\right)\cdot\frac{1}{2} = -\frac{1}{2}\cos\left(\frac{x}{2}\right)$$

18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 $\boxed{\text{(C) } -2\sin(2x)}$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

$y = \cos 2x$

$y' = -\sin(2x)\cdot 2$

$2\cos x\cdot(-\sin x)$

$-2\sin x\cos x$

$= -4\sin x\cos x$

$= -2\cdot \underbrace{2\sin x\cos x}$

$= -2\sin(2x)$

29. The $\lim_{h\to 0} \dfrac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 $\quad$ $\boxed{\text{(B)} \quad 3\sec^2(3x)}$ $\quad$ (C) $\sec^2(3x)$ $\quad$ (D) $3\cot(3x)$ $\quad$ (E) nonexistent

$f(x) = \tan(3x)$

$f'(x) = \sec^2(3x) \cdot 3$

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

$\cos\left(e^{-x}\right) \cdot e^{-x}(-1)$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

$\boxed{\text{(E)}}$ $-e^{-x}\cos(e^{-x})$

8. If $y = \sin x$ and $y^{(n)}$ means "the nth derivative of y with respect to x," then the smallest positive integer n for which $y^{(n)} = y$ is

(A) 2 $\boxed{\text{(B) } 4}$ (C) 5 (D) 6 (E) 8

$y = \sin x$

$y' = \cos x$

$y'' = -\sin x$

$y^{(3)} = -\cos x$

$y^{(4)} = -(-\sin x) = \sin x$

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

$y' = -\sin(2x) \cdot 2$ $\quad y = \cos\left(2\left(\frac{\pi}{4}\right)\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$y'\left(\frac{\pi}{4}\right) = -2\sin\left(2 \cdot \frac{\pi}{4}\right)$

$= -2$

$y - 0 = -2\left(x - \frac{\pi}{4}\right)$

15. If $f(x) = e^{\tan^2 x}$, then $f'(x) =$ $e^{(\tan x)^2} \cdot 2\tan x \cdot \sec^2 x$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

$f = e^{(\tan x)^2}$

7. $\frac{d}{dx}\cos^2(x^3) =$

$\left[\left(\cos(x^3)\right)^2\right]'$

(A) $6x^2 \sin(x^3)\cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$ $= 2\cos(x^3)\left(-\sin(x^3)\right)3x^2$

(E) $-2\sin(x^3)\cos(x^3)$

$$\sin^2 x = (\sin x)^2 \qquad (\sin x)^{-1} = \frac{1}{\sin x}$$

$$\sin^{-1} x \neq (\sin x)^{-1}$$

$$= \arcsin x$$