①

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

②

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$(f(x))^2$

8. If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

(A) 0 (B) 1 (C) $-4f(x)g(x)$

(D) $(-g(x))^2 - (f(x))^2$ (E) $-2(-g(x) + f(x))$

$$h'(x) = 2f(x)f'(x) - 2g(x)g'(x)$$

$$= 2f(x)(-g(x)) - 2g(x)f(x)$$

$$= -4f(x)(g(x))$$

19. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x) + f''(x)$ (B) $f'(x) + \left(f''(x)\right)^2$ (C) $\left(f'(x) + f''(x)\right)^2$

(D) $\left(f'(x)\right)^2 + f''(x)$ (E) $2f'(x) + f''(x)$

$$g'(x) = \underbrace{e^{f(x)}} \cdot \underbrace{f'(x)}$$

$$g''(x) = e^{f(x)} \cdot f'(x) \cdot f'(x) + f''(x)\, e^{f(x)}$$

$$= e^{f(x)} \underbrace{\left(\left(f'(x)\right)^2 + f''(x) \right)}_{h(x)}$$

32. An equation of the line normal to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

~~(A)~~ $4x + y = -10$ ~~(B)~~ $x - 4y = 23$ ~~(C)~~ $4x - y = 2$ ~~(D)~~ $x + 4y = 25$ (E) $x + 4y = -25$

$-1 + 4(-6) = -25$

$$y' = 3x^2 + 6x + 7 \Big|_{x=-1} = 3 - 6 + 7 = 4$$

$$m_N = -\frac{1}{4}$$

$$y = -1 + 3 - 7 - 1 = -6$$

39. Let f and g be differentiable functions such that

$f(1) = 2$, $f'(1) = 3$, $f'(2) = -4$,

$g(1) = 2$, $g'(1) = -3$, $g'(2) = 5$.

If $h(x) = f(g(x))$, then $h'(1) =$

(A) -9 (B) -4 (C) 0 (D) 12 (E) 15

$$h'(x) = f'(g(x))\,g'(x)\Big|_{x=1} = f'(g(1))\,g'(1)$$

$$= f'(2)(-3)$$

$$= (-4)(-3)$$

$$= 12$$

11. Let $y = x\sqrt{1+x^2}$. When $x = 0$ and $dx = 2$, the value of dy is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

$$y = x(1+x^2)^{\frac{1}{2}}$$

when $x=0$

$$y' = 1\cdot(1+x^2)^{\frac{1}{2}} + \cancel{\tfrac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x)x}$$

$$\frac{dy}{dx} = 1\cdot(1+0)^{\frac{1}{2}}$$

$$\frac{dy}{2} = 1 \rightarrow dy = 2$$

Find y'

$$y = e^{\sqrt{x^2+1}} \xrightarrow{} (x^2+1)^{\frac{1}{2}}$$

$$y' = e^{\sqrt{x^2+1}} \, \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)$$

$$= \frac{e^{\sqrt{x^2+1}} \cdot x}{\sqrt{x^2+1}}$$