5)

g) $\lim_{x\to 0}\frac{1-\cos 2x}{x} = \lim_{x\to 0}\frac{1-1+2\sin^2 x}{x}$

* $\cos 2x = 1-2\sin^2 x$

$$= \lim_{x\to 0}\frac{2\,\overset{1}{\boxed{\sin x}}\,\sin x}{\boxed{x}} = 0$$

* $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

$$\lim_{x\to 0}\frac{2(1-\cos 2x)}{2x} = \lim_{2x\to 0}\frac{2(1-\cos 2x)}{2x}$$

How about $2x\to 0$

$= 2\cdot 0 = 0$

e) $\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}-x\right)}{x} = \lim_{x\to 0}\frac{\overset{0}{\cancel{\cos\frac{\pi}{2}\cos x}}+\overset{1}{\cancel{\sin\frac{\pi}{2}}}\sin x}{x}$

$$= \lim_{x\to 0}\frac{\sin x}{x} = 1$$

* $\cos(A-B) = \cos A\cos B + \sin A\sin B$

i) $\lim\limits_{x\to0}\frac{\cos 2x-1}{2x^2} = \lim\limits_{x\to0}\frac{\cancel{1}-2\sin^2 x-\cancel{1}}{2x^2}$

$$= \lim_{x\to0}\overset{\nearrow 1}{\boxed{\frac{-2}{2}}}\cdot\overset{\nearrow 1}{\frac{\sin x}{x}}\cdot\overset{\nearrow 1}{\boxed{\frac{\sin x}{x}}} = -1$$

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4. a) Evaluate $\lim\limits_{x\to0}\frac{\sin x}{x+\sin x}$. $=\frac{1}{2}$

$\downarrow$ near $x=0$

$\sim \frac{x}{x+x}$ $\quad x \approx \sin x \approx \tan x$

$$\lim_{x\to0}\frac{1}{\frac{x+\sin x}{\sin x}} = \lim_{x\to0}\frac{1}{\frac{x}{\sin x}+\frac{\sin x}{\sin x}}$$

$$=\frac{1}{2}$$

f) $$\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{(\tan x)(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{1-\cos^2 x}{\tan x\,(1+\cos x)}=\lim_{x\to 0}\frac{\cancel{\sin x}\sin x}{\frac{\cancel{\sin x}}{\cos x}(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{\sin x\cos x}{1+\cos x}=0$$

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2. **a)** Evaluate $\lim_{x\to 0}\frac{x}{\sin\frac{x}{2}}$.

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\downarrow$$

$$\lim_{x\to 0}\frac{x}{\sin x}=1$$

$$\lim_{x\to 0}\underbrace{\boxed{\frac{\frac{x}{2}}{\sin\frac{x}{2}}}}_{1}\cdot 2=2$$

b) $\lim_{x\to 0}\frac{\sin 3x}{5} = \frac{0}{5} = 0$