

Basic trig limits.

$$1) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 3} \frac{\sin(x-3)}{x-3} = 1$$

$$\lim_{x \rightarrow 0^+} \frac{\sin \sqrt{x}}{\sqrt{x}} = 1$$

$$2) \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \frac{\sin x}{1 + \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x} = 0$$

$$3) \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} = 1$$

near  $x \rightarrow 0$

$\tan x \sim \sin x$

$\sim x$

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$\lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \frac{a}{b}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^+} \frac{\cos x}{x} = +\infty \\ \lim_{x \rightarrow 0^-} \frac{\cos x}{x} = -\infty \end{array} \right\} \lim_{x \rightarrow 0} \frac{\cos x}{x} = \text{DNE}$$

$\begin{array}{c} \nearrow 1 \\ \ominus \end{array}$

$$\lim_{x \rightarrow 0} \frac{\cos x}{|x|} = \infty$$

$$\begin{aligned} \text{d) } \lim_{x \rightarrow 0} \frac{\sin^3 2x}{\sin^3 3x} &= \lim_{x \rightarrow 0} \frac{\sin 2x \sin 2x \sin 2x}{\sin 3x \sin 3x \sin 3x} \\ &= \left(\frac{2}{3}\right)^3 = \frac{8}{27} \end{aligned}$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{x^3}{\tan^3 2x} = \frac{1}{8}$$

$$h) \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x \cos x}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{x \cos x}$$

$$\lim_{x \rightarrow 0} \frac{2 \sin x (1 - \cos x)}{x \cdot \cos x} = 0$$

↑ 1  
↑ 0  
↓ 1