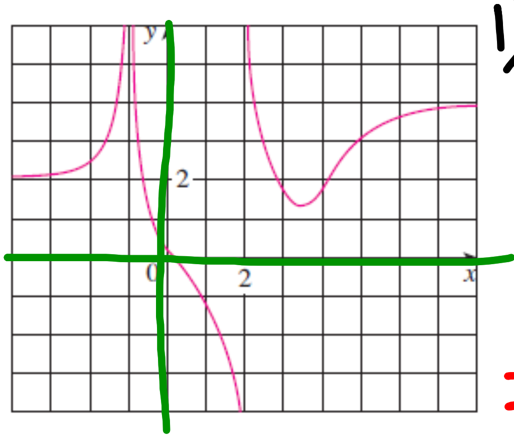

FIGURE 5

1) Hor. Asymp.

$$\left.\lim_{x\to\pm\infty} f(x) = L\right\} \; y = L$$

2) Vert. Asymp.

$$\left.\lim_{x\to a^{\pm}} f(x) = \pm\infty\right\} \; x = a$$

$$\left.\begin{array}{l}\lim_{x\to\infty} f(x) = 4 \\ \lim_{x\to-\infty} f(x) = 2\end{array}\right\} \text{HA}$$

$$\lim_{x\to -1} f(x) = \infty \to \text{VA} \Rightarrow x = -1$$

$$\left.\lim_{x\to 2^{+}} f(x) = \infty \;\&\; \lim_{x\to 2^{-}} f(x) = -\infty\right\} \begin{array}{l}\text{VA} \\ \Rightarrow x = 2\end{array}$$

Continuity.

$y=f(x)$ is continuous at $x=a$,

if $\lim_{x \to a} f(x) = f(a)$

Free Response 2011 #6 Non-Calculator Active

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

To show continuity

$$\lim_{x \to 0} f(x) = f(0)$$

$$\left.\begin{aligned} \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} e^{-4x} = 1 \\ \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} 1 - 2\sin x = 1 \end{aligned}\right\} \lim_{x \to 0} f(x) = 1$$

$$f(0) = 1 - 2\sin(0) = 1$$

Since $\lim_{x \to 0} f(x) = f(0)$, $f(x) = y$ is continuous at $x = 0$

$$f(x) = \begin{cases} 2x+b, & x \geq 3 \\ x^2-1, & x < 3 \end{cases}$$

$y = f(x)$ is cont. at $x=3$.

Find b.

$\lim_{x \to 3} f(x)$ $\quad \lim_{x \to 3^+} f(x)$ $\quad \lim_{x \to 3^-} f(x)$

$2(3)+b=8 \quad 3^2-1=8$

$b=2$

Free Response 2011B #2 Calculator Active

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

$$\lim_{t \to 5^+} r(t) = 1000e^{-.2(5)} = 367.879$$

$$\lim_{t \to 5^-} r(t) = \frac{600(5)}{5+3} = 375$$

since $\lim_{t \to 5^+} r(t) \neq \lim_{t \to 5^-} r(t)$, $\lim_{t \to 5} r(t) = \text{DNE}$.

So, this shows that $y = r(t)$ is not cont at $x = 5$.