$$\lim_{x \to a} f(x) = L$$

$\forall\ \varepsilon > 0, \exists\ \delta > 0$

if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$

$$\lim_{x \to a} \frac{f(x)}{g(x)} \rightarrow \begin{matrix} f(a) = 0 \\ g(a) = 0 \end{matrix}$$

① factor, cancel

② mult. by conjugate

③ complex fraction, mult. by common deno

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1}$$

$$x^3-y^3=(x-y)(x^2+xy+y^2)$$

$$\frac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}} = \frac{x^2+x+1}{x+1}$$

$$\frac{3}{2}$$

$$\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3} \cdot \frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$$

$$= \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)} = \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+6}+3)}$$

$$= \frac{1}{\sqrt{3+6}+3} = \frac{1}{6}$$

$$\lim_{h\to 0} \frac{\left(\frac{1}{4+h} - \frac{1}{4}\right)(4+h)4}{(h)(4+h)4}$$

$$= \lim_{h\to 0} \frac{4-4-h}{h(4+h)4} = \lim_{h\to 0} \frac{-h}{h(4+h)4}$$

$$= \lim_{h\to 0} \frac{-1}{(4+h)4} = \frac{-1}{16}$$

18. $\lim_{x\to 1} \frac{x^2-4}{x-1} = DNE$

$$\lim_{x\to 1^+} \frac{x^2-4}{x-1} = -\infty$$

$*$ Let $x = 1.1$

$$\frac{1.21-4}{1.1-1} \quad \frac{-}{+} \to -\infty$$

$$\lim_{x\to 1^-} \frac{x^2-4}{x-1} = +\infty$$

Let $x = .9$

$$\frac{.9^2-4}{.9-1} \Rightarrow \frac{-}{-}$$

21. $\lim_{x \to \infty} \frac{5x + \sin x}{x}$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

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