84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

$\frac{dy}{dt} = ky$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + c$

$|y| = e^{kt+c} = Ce^{kt}$

$y = Ce^{kt}$

t	y
0	c
10	2c

$2c = ce^{k(10)}$

$2 = e^{10k}$

$\ln 2 = 10k$

$\frac{\ln 2}{10} = k$

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

$\frac{dy}{dx} = 4y$

$\int \frac{dy}{y} = \int 4\,dx$

$\ln|y| = 4x + C$

$|y| = e^{4x+c} = Ce^{4x}$

$y = Ce^{4x} \rightarrow 4 = Ce^0, C = 4$

23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

$\int y\,dy = \int \frac{-x}{e^{x^2}}dx = \frac{1}{2}\int -2x e^{-x^2}dx$

$u = -x^2$
$du = -2x\,dx$

$\frac{1}{2}\int e^u du$

$= \frac{1}{2}e^u + C$

$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} + C$

$y^2 = e^{-x^2} + C$

$y = \pm\sqrt{e^{-x^2} + C}$

$y = \sqrt{e^{-x^2} + 3}$

$2 = \sqrt{e^0 + c}$

$2 = \sqrt{1 + c} \to c = 3$

43. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

$\ln 27 - \ln 2$

$3\ln 3$ $\frac{\ln 2}{3}t$

$\frac{dP}{dt} = kP$

$\frac{dP}{P} = k\,dt$

$P = ce$

$\ln|P| = kt + c$

t	P
0	c
3	2c
t	3c

$P = Ce^{kt}$

$2C = Ce^{3k}$

$2 = e^{3k}$

$3\cancel{c} = \cancel{c}e^{\frac{\ln 2}{3}t}$ $t = \frac{3\ln 3}{\ln 2}$

$\ln 2 = 3k$ $\ln 3 = \frac{\ln 2}{3}t$

$\frac{\ln 2}{3} = k$

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

$\frac{dy}{dx} = -y$ $\quad \frac{dy}{y} = -dx$ $\quad \ln|y| = -x + c$

$y = Ce^{-x} \rightarrow y = e^{1} \cdot e^{-x} = e^{1-x}$

$1 = ce^{-1} = \frac{c}{e}$

33. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

$y = Ce^{-2t} \rightarrow y = e^{-2t} \rightarrow \frac{1}{2} = e^{-2t}$

$1 = ce^{0}$

$\ln\frac{1}{2} = -2t$

$\frac{\ln 2}{2} = \frac{-\ln 2}{-2} = \frac{\ln\frac{1}{2}}{-2} = t$

$A = Pe^{rt}$

rate

$10\% \rightarrow .1$