57. If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, find $F''(2)$.

$$f'(t) = \left(\frac{\sqrt{1+(t^2)^4}}{t^{\cancel{2}}}\right)2\cancel{t}$$

$$F'(x) = f(x)$$

$$F''(x) = f'(x) = \frac{2\sqrt{1+x^8}}{x}$$

$$F''(2) = f'(2) = \frac{\cancel{2}\sqrt{1+2^8}}{\cancel{2}} = \sqrt{257}$$

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53. $g(x) = \int_{2x}^{3x} \underbrace{\frac{u^2 - 1}{u^2 + 1}}_{h(u)} \, du = H(u)\Big]_{2x}^{3x}$

Find $\frac{dg}{dx}$.

$\left[\textit{Hint}: \int_{2x}^{3x} f(u)\, du = \int_{2x}^{0} f(u)\, du + \int_{0}^{3x} f(u)\, du \right]$

$= H(3x) - H(2x)$

$* \; H(u) = \int h(u)\, du$

$\frac{dg}{dx} = h(3x) \cdot 3 - h(2x) \cdot 2$

$= \frac{3\left((3x)^2 - 1\right)}{(3x)^2 + 1} - \frac{2\left((2x)^2 - 1\right)}{(2x)^2 + 1}$

55. $y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t \, dt$

$$\frac{dy}{dx} = \sqrt{x^3}\sin(x^3)(3x^2) - \sqrt{\sqrt{x}}\,\sin\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)$$

$\sqrt{x^3} \to x^{\frac{3}{2}}$ $\sqrt{\sqrt{x}} \to x^{\frac{1}{4}}$ $\sqrt{x} \to x^{\frac{1}{2}}$

$$= 3x^{7/2}\sin(x^3) - \frac{1}{2\sqrt[4]{x}}\sin\sqrt{x}$$

17. $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\,du$

$$\frac{dy}{dx} = +3\left(\frac{+(1-3x)^3}{1+(1-3x)^2}\right) = \frac{3(1-3x)^3}{1+(1-3x)^2}$$