

57. If $F(x) = \int_1^x f(t) dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du$,
find $F''(2)$.

$$F'(x) = f(x)$$

$$f'(t) = \left(\frac{\sqrt{1+(t^2)^4}}{t^2} \right) \cdot 2t$$

$$F''(x) = f'(x) = \frac{2\sqrt{1+x^8}}{x}$$

$$F''(2) = f'(2) = \frac{2\sqrt{1+2^8}}{2} = \sqrt{257}$$

53. $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1} du = H(u) \Big|_{2x}^{3x}$

[Hint: $\int_{2x}^{3x} f(u) du = \int_{2x}^0 f(u) du + \int_0^{3x} f(u) du$]

Find $\frac{dg}{dx}$.

$$= H(3x) - H(2x)$$

$$* H(u) = \int h(u) du$$

$$\frac{dg}{dx} = h(3x) \cdot 3 - h(2x) \cdot 2$$

$$= \frac{3((3x)^2 - 1)}{(3x)^2 + 1} - \frac{2((2x)^2 - 1)}{(2x)^2 + 1}$$

$$55. y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t \, dt$$

$$\begin{aligned} \frac{dy}{dx} &= \sqrt{x^3} \sin(x^3) (3x^2) - \sqrt{\sqrt{x}} \sin \sqrt{x} \left(\frac{1}{2\sqrt{x}} \right) \\ &= 3x^{7/2} \sin(x^3) - \frac{1}{2\sqrt{x}} \sin \sqrt{x} \end{aligned}$$

Handwritten annotations in purple:

- An arrow points from x^3 in the first term to x^3 written above it.
- An arrow points from $\sqrt{x^3}$ to $x^{3/2}$ written below it.
- A circle is drawn around $\frac{1}{2\sqrt{x}}$ in the second term, with an arrow pointing to $x^{-1/2}$ written below it.

$$17. y = \int_{1-3x}^1 \frac{u^3}{1+u^2} du$$

$$\frac{dy}{dx} = +3 \left(\frac{+(1-3x)^3}{1+(1-3x)^2} \right) = \frac{3(1-3x)^3}{1+(1-3x)^2}$$