Fun. thm. of calculus

Let $g(x) = \int_2^x r+3\,dr = \frac{r^2}{2} + 3r\Big]_2^x$

$= \left(\frac{x^2}{2} + 3x\right) - \left(\frac{2^2}{2} + 3(2)\right)$

1) Find $g(x)$ in terms of x.

2) Find $\frac{dg}{dx}$

$g(x) = \frac{x^2}{2} + 3x - 8$

$\frac{dg}{dx} = x+3$

Let $g(x) = \int_{\frac{\pi}{3}}^x \cos r\,dr$

Find $\frac{dg}{dx}$.

$= \sin r\Big]_{\frac{\pi}{3}}^x = \sin x - \sin\frac{\pi}{3}$

$\frac{dg}{dx} = \cos x \leftarrow g(x) = \sin x - \frac{\sqrt{3}}{2}$

Let $f(x) = \int_a^x g(r)\,dr$

$\frac{df}{dx} = g(x)$

1) $f(x)=\int_3^x 2r^3-\cos(r^2)\,dr$

Find $\frac{df}{dx}=2x^3-\cos(x^2)$

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2) $f(x)=\int_x^3 \underbrace{2r^3-\cos(r^2)}_{g(r)}\,dr = G(r)\Big]_x^3$

Find $\frac{df}{dx}=-2x^3+\cos(x^2)$

$=G(3)-G(x)$

$\Rightarrow -g(x)$

Let $f(x)=\int_3^{2x} \underbrace{e^{2r}-\frac{1}{r}}_{g(r)}\,dr$ $\rightarrow e^{4x}-\frac{2}{x}$

Find $\frac{df}{dx}=2e^{2x}-\frac{2}{x}$ $\rightarrow 4e^{4x}-\frac{1}{x}$

$=e^{2x}-\frac{1}{x}$ $\rightarrow e^{4x}-\frac{1}{2x}$

$G(r)\Big]_3^{2x}=G(2x)-G(3)=f(x)$

$\left(e^{4x}-\frac{1}{2x}\right)2=g(2x)\cdot 2 \;=\frac{df}{dx}$

Let $f(x) = \int_a^{g(x)} h(r)\,dr$

$$\frac{df}{dx} = h(g(x))\frac{dg}{dx}$$

Let $f(x) = \int_3^{x^2} \sqrt{r - \cos r}\,dr$

$$\frac{df}{dx} = \left(\sqrt{x^2 - \cos(x^2)}\right)(2x)$$