

Area under  $y = 2x + 1$

n	area
3	108
4	120
6	132
12	144

$$A = \frac{(1+25)12}{2}$$

$$= \boxed{156}$$

$$n \left[ \frac{12}{n} \left[ f(0) + f\left(\frac{12}{n}\right) + f\left(2\left(\frac{12}{n}\right)\right) \right. \right.$$

$$\left. + f\left(3\left(\frac{12}{n}\right)\right) + f\left(4\left(\frac{12}{n}\right)\right) + \dots \right]$$

$$\frac{12}{n} \left( \boxed{2\left(0 \cdot \frac{12}{n}\right) + 1} + \boxed{2\left(1 \cdot \frac{12}{n}\right) + 1} + \boxed{2\left(2 \cdot \frac{12}{n}\right) + 1} + \dots \right)$$

$$\frac{12}{n} \left( \frac{(0+n-1)\frac{12}{n} + n}{2} \right)$$

$2\left((n-1)\frac{12}{n}\right)$   $\nearrow$   $n^{\text{th}}$  term

$$\frac{12}{n} \left( \frac{12n-12}{2} + n \right) = \frac{12}{n} (6n-6+n)$$

$$= \frac{12}{n} (7n-6)$$

$$y = 2x + 1$$

$$0 + 1$$

$$+ 2\left(\frac{a}{n}\right) + 1$$

$$+ 2\left(\frac{2a}{n}\right) + 1$$

$$+ 2\left(\frac{3a}{n}\right) + 1$$

$$\vdots$$

$$+ 2\left(\frac{(n-1)a}{n}\right) + 1$$

$$= \frac{a}{n} \left[ \left(1 + 2\frac{(n-1)a}{n} + 1\right) n \right]$$

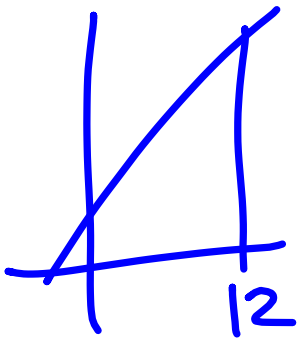
$$= a \left( \frac{1 + 2\frac{(n-1)a}{n} + 1}{n} \right) = \frac{a^2(n-1)}{n}$$

$$= \frac{a^2 n - a^2}{n} + a$$

$$= a^2 - \frac{a^2}{n} + a$$

as  $n \rightarrow \infty$

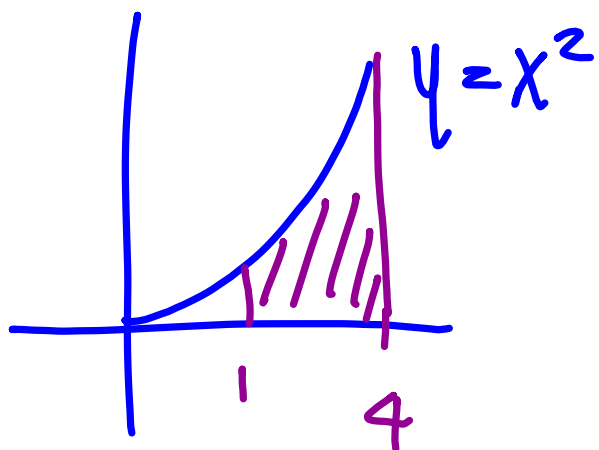
$$= a^2 + a \rightarrow \underline{X^2 + X}$$



$$\int_0^{12} 2x+1 dx = \text{Area under}$$

$y = 2x+1$

$$x^2+x \Big|_0^{12} = (2^2+(12)) - (0^2+0)$$
$$= 156$$



$$\int_1^4 x^2 dx = \left. \frac{x^3}{3} \right|_1^4$$
$$= \frac{4^3}{3} - \frac{1^3}{3} = \frac{63}{3} = 21$$

