43. $\int \frac{1+x}{1+x^2}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$= \int \frac{1}{1+x^2} + \frac{x}{1+x^2}\,dx$

$= \arctan x + \frac{1}{2}\int \frac{1}{u}\,du = \arctan x + \frac{1}{2}\ln|u| + C$

$= \arctan x + \frac{1}{2}\ln(1+x^2) + C$

45. $\displaystyle\int \frac{x}{\sqrt[4]{x+2}}\,dx$

$u = x+2 \rightarrow x = u-2$

$du = dx$

$$\int \frac{u-2}{\sqrt[4]{u}}\,du = \int \frac{u}{u^{1/4}} - \frac{2}{u^{1/4}}\,du$$

$$= \int u^{3/4} - 2u^{-1/4}\,du$$

$$= \frac{4}{7}u^{7/4} - 2\left(\frac{4}{3}\right)u^{3/4} + C$$

$$= \frac{4}{7}(x+2)^{7/4} - \frac{8}{3}(x+2)^{3/4} + C$$

31. $\int \frac{\cos x}{\sin^2 x}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int \frac{du}{u^2} = \int u^{-2}\,du$

$= -u^{-1} + C$

$= -\frac{1}{\sin x} + C$

$u = \sin^2 x$

$du = 2\sin x \cos x\,dx$

29. $\int e^{\tan x} \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int e^u \, du = e^u + C$

$= e^{\tan x} + C$

35. $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx$

$\sin 2x \rightsquigarrow 2\sin x\cos x$

$u = \cos x$

$du = -\sin x\,dx$

$= -\displaystyle\int \frac{du}{u}$

$= -\ln|u| + C$

$= -\ln|1+\cos^2 x| + C$

$u = 1+\cos^2 x$

$du = 2\cos x(-\sin x)\,dx$

41. $\int \frac{dx}{\sqrt{1-x^2}\sin^{-1}x}$

$u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int \frac{du}{u} = \ln|u| + C$

$= \ln|\sin^{-1}x| + C$

39. $\int \sec^3 x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int \sec^2 x \cdot \sec x \tan x \, dx$

$\int u^2 \, du = \frac{u^3}{3} + C = \frac{\sec^3 x}{3} + C$