8.

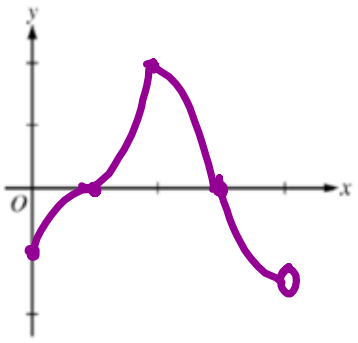
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



a) rel max
x=2
b/c f' + → −

9. Consider the curve given by $y^2 = 2 + xy$

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has a slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

a) $2yy' = y + xy'$ $\quad y' = \frac{dy}{dx} = \frac{y}{2y-x}$

$2yy' - xy' = y$

$y'(2y-x) = y$

b) $\frac{1}{2} = \frac{y}{2y-x}$ $\quad y^2 = 2 + xy \Big|_{x=0}$

$2y = 2y - x$ $\quad y^2 = 2$

$x = 0$ $\quad y = \pm\sqrt{2}$

$(0, \sqrt{2}), (0, -\sqrt{2})$

c) hor. $\rightarrow \frac{dy}{dx} = 0 = \frac{y}{2y-x} \rightarrow y = 0$

$y^2 = 2 + xy \Big|_{y=0}$ $\quad 0 = 2 + x(0)$

$x = \{\}$

So, there is no pt w/ hor. tangent.

12. The position of a particle moving on the x-axis at time $t > 0$ seconds is: $x(t) = e^t - \sqrt{t}$ feet.

(a) Find the average velocity of the particle over the interval $1 \le t \le 3$.

(b) In what direction and how fast is the particle moving at $t = 1$ seconds?

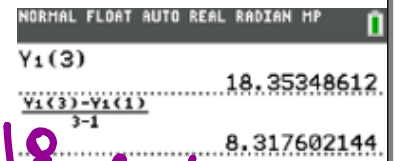
(c) When is the particle moving to the right?

(d) Find the position of the particle when its velocity is zero.

ARC

Slope

a) $\dfrac{x(3) - x(1)}{3 - 1} = 8.318$ ft/sec



d) $y^2 = 2 + xy$

$y = 3 \qquad t = 5$

$\frac{dy}{dt} = 6 \qquad \frac{dx}{dt} = ?$

$3^2 = 2 + x(3)$

$7 = 3x$

$\frac{7}{3} = x$

$2y\frac{dy}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$

$2(3)(6) = \frac{dx}{dt}(3) + \frac{7}{3}(6)$

$36 = 3\frac{dx}{dt} + 14$

$\frac{22}{3} = \frac{dx}{dt}$

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