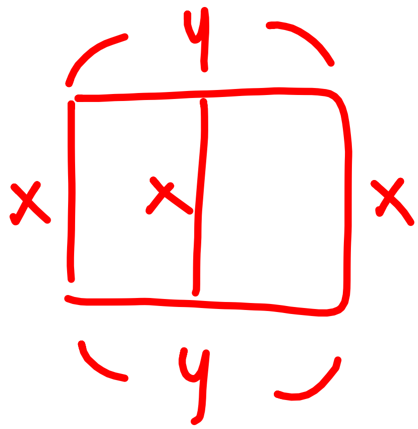
11. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?



$xy = 1.5 \text{ mil ft}^2$

$L = 3x + 2y$ $\quad y = \dfrac{1{,}500{,}000}{x}$

$= 3x + \dfrac{3000000}{x}$

$L' = 3 - \dfrac{3000000}{x^2} = 0$

$3 = \dfrac{3000000}{x^2}$

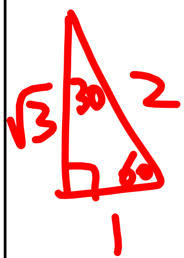
$3x^2 = 3000000$

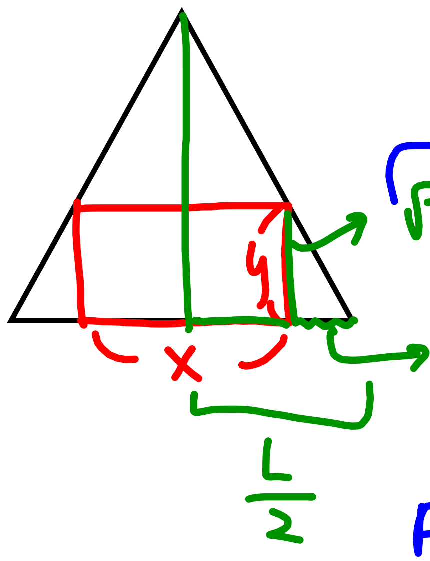
$\underline{x = 1000}$

$y = \dfrac{1{,}500{,}000}{1000}$

$\underline{y = 1500}$

23. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.





$$A = xy$$

$$\sqrt{3}\left(\frac{L}{2} - \frac{x}{2}\right) = y$$

$$\frac{L}{2} - \frac{x}{2}$$

$$\frac{\sqrt{3}}{2}x$$

$$A = x\left(\frac{\sqrt{3}}{2}(L - x)\right)$$

$$= \frac{\sqrt{3}}{2}Lx - \frac{\sqrt{3}}{2}x^2$$

$$A' = \frac{\sqrt{3}}{2}L - \sqrt{3}x = 0$$

$$\frac{\cancel{\sqrt{3}}}{2}L = \cancel{\sqrt{3}}x$$

$$\frac{L}{2} = x$$

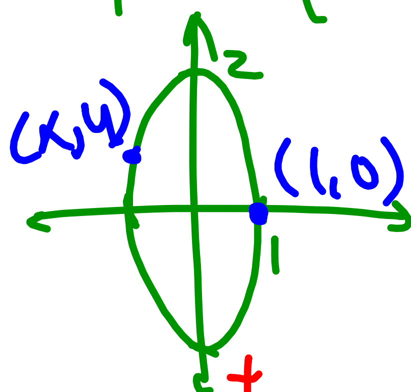
$$y = \sqrt{3}\left(\frac{L}{2} - \frac{x}{2}\right)$$

$$= \sqrt{3}\left(\frac{L}{2} - \frac{L}{4}\right) = \frac{\sqrt{3}}{4}L = y$$

19. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (1, 0).

$\to y^2 = 4 - 4x^2$

$\to \frac{x^2}{1} + \frac{y^2}{4} = 1$



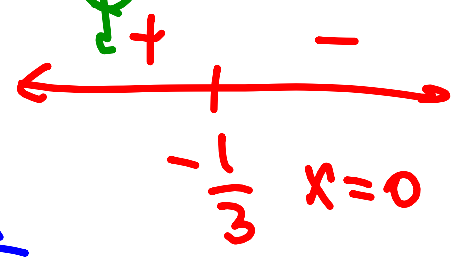
$$d = \sqrt{(x-1)^2 + y^2}$$

$$= \sqrt{(x-1)^2 + 4 - 4x^2}$$

$$D = d^2 = (x-1)^2 + 4 - 4x^2$$

$$D' = 2(x-1) - 8x$$

$$= -6x - 2 = 0 \to x = -\frac{1}{3}$$



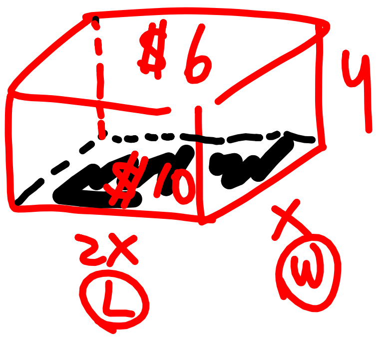
$$y^2 = 4 - 4\left(-\tfrac{1}{3}\right)^2 \qquad y = \pm\frac{4\sqrt{2}}{3}$$

$$= 4 - \frac{4}{9} = \frac{32}{9}$$

$$\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right), \left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$$

14. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

15. Do Exercise 14 assuming the container has a lid that is made from the same material as the sides.



$$V = 10 = 2x^2 y$$

$$y = \frac{5}{x^2}$$

$$c = \underline{(2x^2)}10 + (\underbrace{xy\cdot 2 + 2xy \cdot 2}\ \underbrace{+2x^2})6$$

$$C = 20x^2 + 6\left(2\cancel{x}\frac{5}{\cancel{x^2}_{x}} + 4\cancel{x}\cdot\frac{5}{\cancel{x^2}_{x}} + 2x^2\right)$$

$$\frac{10}{x} \qquad \frac{20}{x}$$

$$= 20x^2 + \frac{180}{x} + 12x^2$$

$$c' = 64x - \frac{180}{x^2} = 0$$

$$64x = \frac{180}{x^2}$$

$$64x^3 = 180 \rightarrow x^3 = \frac{180}{64}$$

$$x = \frac{\sqrt[3]{180}}{4}$$

10 → 2, 5

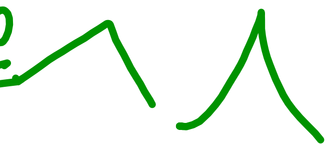
18 → 2, 3, 3

MVT

When $y=f(x)$ is cont. & differentiable

for $a \le x \le b$

No

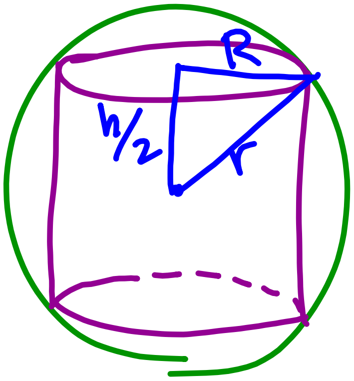


$$\frac{f(b)-f(a)}{b-a} = f'(c), \quad a < c < b$$

* $f(a)$ doesn't have to be = to $f(b)$

** Roll's Thm, $f(a)=f(b)$

27. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.



$$V = \pi R^2 h$$

$$\left(\frac{h}{2}\right)^2 + R^2 = r^2$$

$$R^2 = r^2 - \frac{h^2}{4}$$

$$V = \pi\left(r^2 - \frac{h^2}{4}\right)h$$

$$V = \pi r^2 h - \frac{\pi h^3}{4}$$

$$V' = \pi r^2 - \frac{3\pi h^2}{4} = 0$$

$$\cancel{\pi} r^2 = \frac{3\cancel{\pi} h^2}{4}$$

$$\frac{4r^2}{3} = h^2 \rightarrow h = \frac{2r}{\sqrt{3}}$$

$$V = \pi\left(r^2 - \frac{r^2}{3}\right)\frac{2r}{\sqrt{3}}$$

$$= \pi \frac{2}{3} r^2 \left(\frac{2r}{\sqrt{3}}\right) = \frac{\pi 4}{3\sqrt{3}} r^3$$

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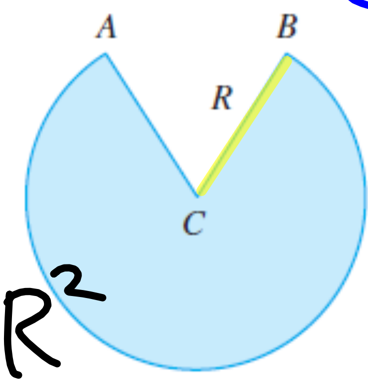
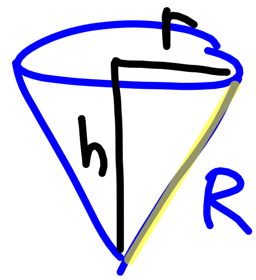
37. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

$$V=\frac{1}{3}\pi r^2 h$$





$$r^2+h^2=R^2$$

$$r^2=R^2-h^2$$

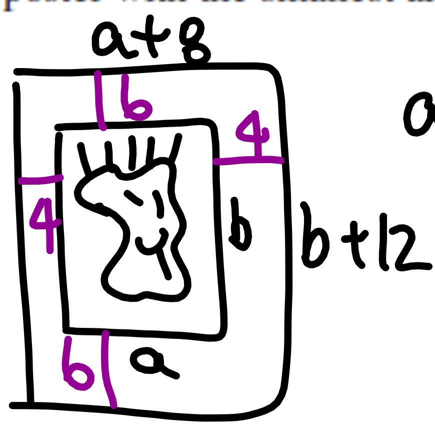
$$V=\frac{1}{3}\pi(R^2-h^2)h$$

$$=\frac{1}{3}\pi R^2 h-\frac{1}{3}\pi h^3$$

$$V'=\frac{\pi R^2}{3}-\pi h^2$$

$$\frac{R^2}{3}=h^2 \longrightarrow h=\frac{R}{\sqrt{3}}$$

31. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area.



$ab = 384$

$b = \frac{384}{a}$

$A = (a+8)(b+12)$

$= (a+8)\left(\frac{384}{a} + 12\right)$

$A = 384 + 12a + \frac{8 \cdot 384}{a} + 96$

$A' = 12 - \frac{8 \cdot 384}{a^2} = 0$

$12 = \frac{8 \cdot 384}{a^2}$

$a = 16 \leftarrow a^2 = \frac{\overset{2}{\cancel{8}} \cdot \overset{128}{\cancel{384}}}{\underset{3}{\cancel{12}}} = 256$

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$$\lim_{x \to 0} \frac{\sin(3x)}{e^x - 1} = \lim_{x \to 0} \frac{3\cos 3x}{e^x} = 3$$