EXAMPLE 6 A store has been selling 200 DVD burners a week at $350 each. A market survey indicates that for each $10 rebate offered to buyers, the number of units sold will increase by 20 a week. Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?

$$R(x) = (350 - 10x)(200 + 20x)$$

$$= -200x^2 - 2000x + 7000x + 70000$$

$$= -200x^2 + 5000x + 70000$$

$$R' = -400x + 5000 = 0$$

$$x = \frac{5000}{400} = 12.5$$

R': $+$ on $x < 12.5$, $-$ on $x > 12.5$

$x = 10$ 12.5

Rebate: $10(12.5)$

$= \$125$

EXAMPLE 4 A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible (see Figure 7). He could row his boat directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible? (We assume that the speed of the water is negligible compared with the speed at which the man rows.)

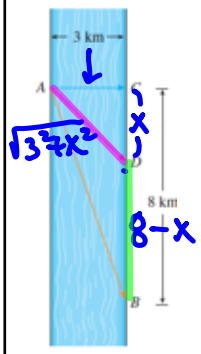


FIGURE 7

min time

$T = \frac{D}{S}$

$$T = \frac{\sqrt{9+x^2}}{6} + \frac{8-x}{8}$$

$$= \frac{1}{6}(9+x^2)^{\frac{1}{2}} + 1 - \frac{1}{8}x$$

$$T' = \frac{1}{6}(9+x^2)^{-\frac{1}{2}}(2x) - \frac{1}{8} = 0$$

$$\rightarrow \frac{x}{6\sqrt{9+x^2}} - \frac{1}{8} = 0$$

$$\frac{x}{6\sqrt{9+x^2}} = \frac{1}{8}$$

$$8x = 6\sqrt{9+x^2}$$

$$4x = 3\sqrt{9+x^2}$$

$$16x^2 = 9(9+x^2) \rightarrow 7x^2 = 81$$

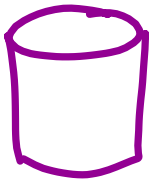
$$x = \sqrt{\frac{81}{7}} \approx 3.4$$

he should row to pt. D, where it is 3.4 km below pt. C. Then, he should run to B.

EXAMPLE 2 A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

1000 cm^3

SA



$S = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

$S = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2000}{r}$

$S' = 4\pi r - \frac{2000}{r^2} = 0$

$4\pi r^3 = 2000$