January 06, 2020

10. The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

$$f' = \frac{4}{3}x^3 - x^4 \rightarrow \text{Find max}$$

$$f'' = 4x^2 - 4x^3$$

$$= \underset{0}{\overset{\oplus}{4x^2}}\underset{1}{\overset{\ominus}{(1-x)}}$$

+ + −

0 1

$x=10$

11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

$3x = 4y$

$\frac{3}{4}x = y$

$m = \frac{3}{4}$

$m_1 = m_2$

share a pt.

$m = y' = 3x^2$

$\frac{3}{4} = 3x^2$

$\frac{1}{4} = x^2$

$y = \frac{3}{4}\left(\frac{1}{2}\right)$

$= \frac{3}{8}$

$x = \frac{1}{2}$

QI

$\leftarrow \pm\frac{1}{2} = x$

$y = x^3 + K$

$\frac{3}{8} = \left(\frac{1}{2}\right)^3 + K$

$\frac{3}{8} = \frac{1}{8} + K$

$\frac{1}{4} = \frac{2}{8} = K$

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

$$\frac{dr}{dt} = .3 \qquad S = 100\pi \qquad \frac{dV}{dt} = ?$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 100\pi(.3)$$

$$= \underline{30\pi}$$

23. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is $\longrightarrow$ $\lim_{h \to 0} \frac{\ln(2+h) - \ln 2}{h}$

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$x = 2$

$f(x) = \ln x$

$f'(x) = \frac{1}{x}\Big|_{x=2} = \frac{1}{2}$

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$

(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

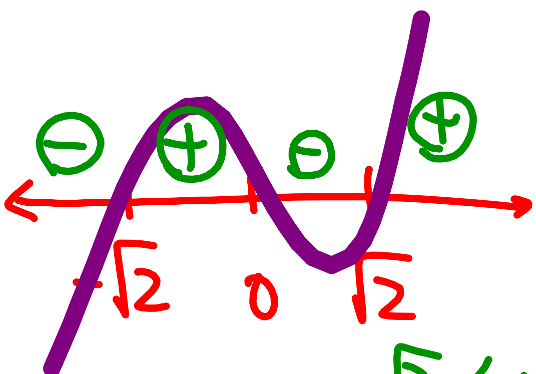
(E) $-2 < x < 2$

$f' = 15x^4 - 60x^2$

$f'' = 60x^3 - 120x$

$= 60x(x^2 - 2)$

$0, \pm\sqrt{2}$



$-\sqrt{2} < x < 0, \sqrt{2} < x$

40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

$$\sec^2(xy)\left(y + x\frac{dy}{dx}\right) = 1$$

$$y + x\frac{dy}{dx} = \cos^2(xy)$$

$$\frac{\cancel{x}\frac{dy}{dx}}{\cancel{x}} = \frac{\cos^2(xy) - y}{x}$$

36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

$y' = e^{nx} \cdot n$

$y'' = e^{nx} \cdot n^2$