Name:___________________________________ Date:____________

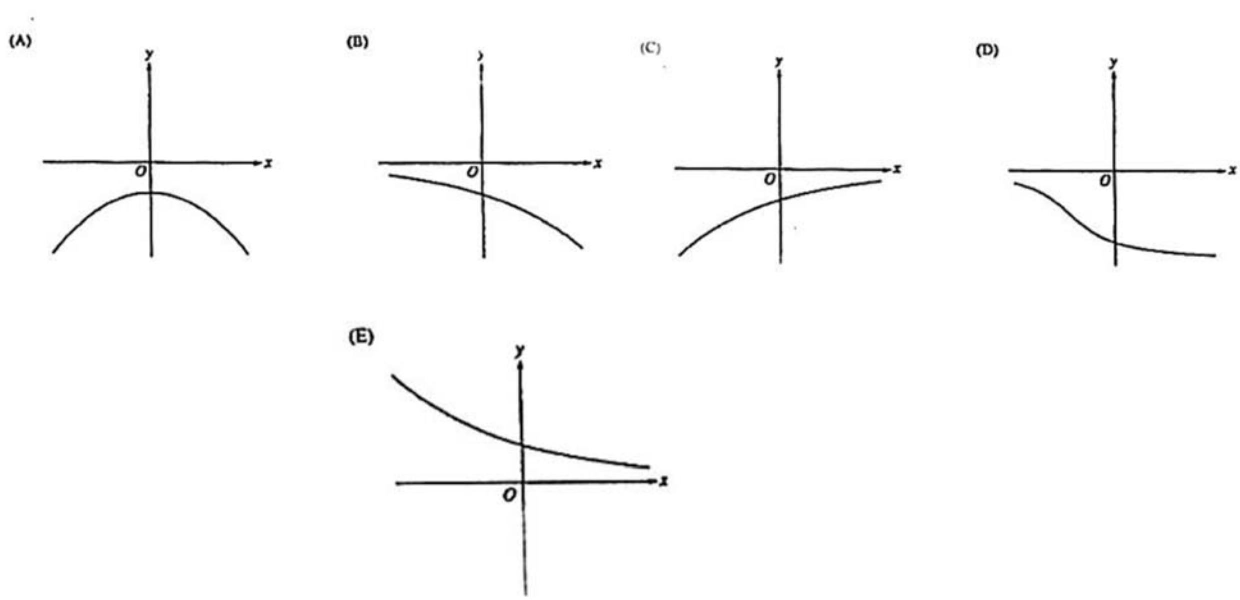
AP Calculus AB Midterm Review

Midterm Exam: Wednesday, January, 22nd , 12:15 – 1:45, in the cafeteria

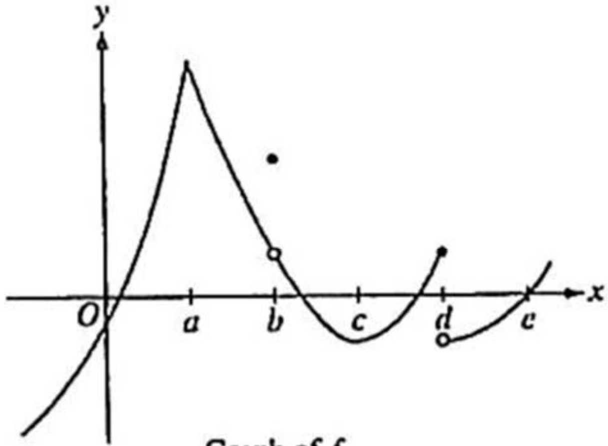
Format:
Part I: 8 multiple choice questions with calculator (4pts each)
Part II: 9 multiple choice questions without calculator (4pts each)
Part III: 3 free response questions (12pts each)

The following sets of questions are not comprehensive; they are a mere sampling of the topics that we have covered this semester. To fully prepare for this exam, review your old exams, your notes and homework assignments.

1. The function *f* has the property that $f(x)$, $f'(x)$ and $f''(x)$ are negative for all real values *x*. Which of the following could be the graph of *f* ?

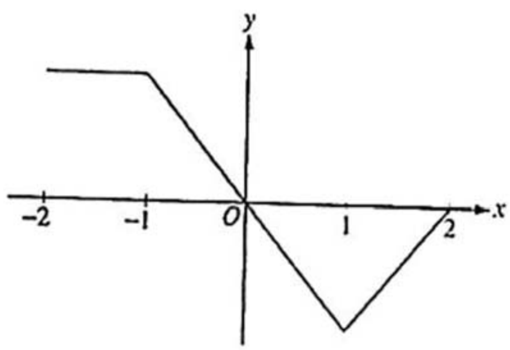


2. The graph of a function *f* is shown below. At which value of *x* is *f* continuous, but not differentiable?
(a) *a* (b) *b* (c) *c* (d) *d* (e) *e*



Graph of *f*

3. The graph of $f'(x)$, the derivative of the function f, is shown below. Which of the following statements is true about f ?
(a) *f* is decreasing for $-1 \le x \le 1$.
(b) f is increasing for $-2 \le x \le 0$.
(c) *f* is increasing for $1 \le x \le 2$.
(d) *f* has a local minimum at $x = 0$.
(e) *f* is not differentiable at $x = -1$ and $x = 1$.



Graph of *f'*

4. $\lim_{h\to 0}\dfrac{(1+h)^6-1}{h}$ is

(a) 0 (b) 1 (c) 6 (d) ∞ (e) nonexistent

5. $\lim_{h\to 0}\dfrac{\sqrt[3]{8+h}-2}{h}$ is

(a) 0 (b) $\dfrac{1}{12}$ (c) 1 (d) 192 (e) ∞

6. $\lim_{h\to 0}\dfrac{\ln(e+h)-1}{h}$ is

(a) 0 (b) $\dfrac{1}{e}$ (c) 1 (d) e (e) nonexistent

7. $\lim_{x\to 0}\dfrac{\cos x-1}{x}$ is

(a) – 1 (b) 0 (c) 1 (d) ∞ (e) none of these

8.

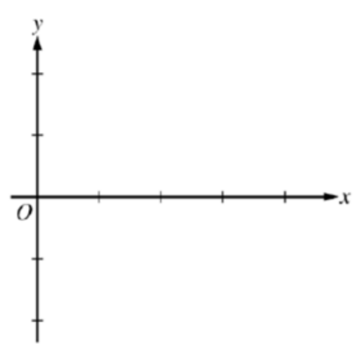
x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



9. Consider the curve given by $y^2 = 2 + xy$

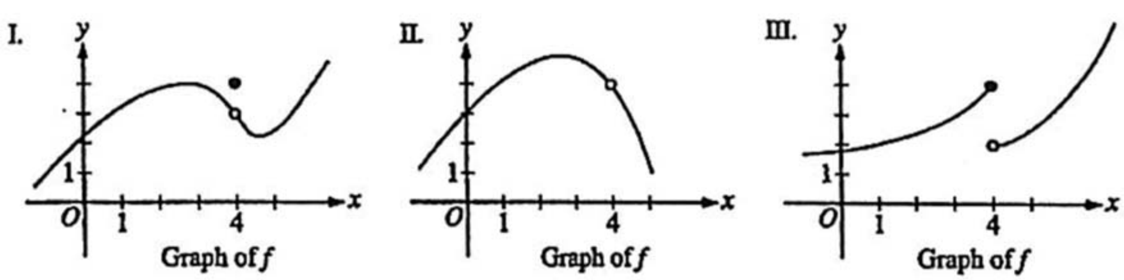
(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has a slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

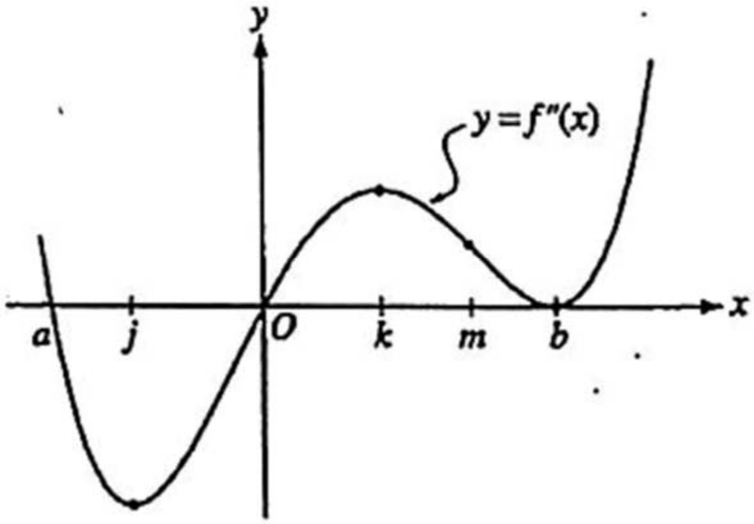
(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

10. For which of the following does $\lim_{x \to 4} f(x)$ exist ?



(a) I only (b) II only (c) III only (d) I and II only (e) I and III only

11. The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of $f''(x)$ is shown below. For what values of x does the graph of f have a point of inflection?



(a) 0 and a only
(b) 0 and m only
(c) b and j only
(d) 0, a and b
(e) b, j and k

Selected Calculator Active Questions

12. The position of a particle moving on the x-axis at time $t > 0$ seconds is: $x(t) = e^t - \sqrt{t}$ feet.
    (a) Find the average velocity of the particle over the interval $1 \leq t \leq 3$.
    (b) In what direction and how fast is the particle moving at $t = 1$ seconds?
    (c) When is the particle moving to the right?
    (d) Find the position of the particle when its velocity is zero.

13. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?
(a) -0.701 (b) -0.567 (c) -0.391 (d) -0.302 (e) -0.258

14. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0,10)?
(a) One (b) Three (c) Four (d) Five (e) Seven

15. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(a) $y = 8x - 5$ (b) $y = x + 7$ (c) $y = x + 0.763$

(d) $y = x - 0.122$ (e) $y = x - 2.146$

16. Let *f* be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of *x* at which the graph of *f* changes concavity?

(a) 0.56 (b) 0.93 (c) 1.18 (d) 2.38 (e) 2.44

17. If $f(x) = \sqrt[5]{x^3 - 2x}$, then $f'(\sqrt{3}) =$

(a) 0.129 (b) 0.902 (c) 0.906 (d) 1.116 (e) 2.173

More Non-calculator Questions

18. Given: $f(x) = 5x^3 + x$. If $g(x) = f^{-1}(x)$, find $g'(6)$.

19. Find the points of discontinuity, if any: $f(x) = \begin{cases} 2x+3 & \text{if} \quad x \le 4 \\ 7 + \dfrac{16}{x} & \text{if} \quad x > 4 \end{cases}$

20. Find a value for the constant *k*, if possible, that will make the function continuous:

(a) $f(x) = \begin{cases} 7x-2 & \text{if} \quad x \le 1 \\ kx^2 & \text{if} \quad x > 1 \end{cases}$

(b) $f(x) = \begin{cases} kx^2 & \text{if} \quad x \le 2 \\ 2x+k & \text{if} \quad x > 2 \end{cases}$

21. $\displaystyle\lim_{x \to +\infty} \sin\left(\frac{\pi x}{2 - 3x}\right)$

22. $\displaystyle\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}$

23. $\lim_{x \to 0} \dfrac{\tan 7x}{\sin 3x}$

24. $\lim_{\theta \to 0} \dfrac{\theta^2}{1 - \cos\theta}$

25. Find a nonzero value for the constant *k* that makes

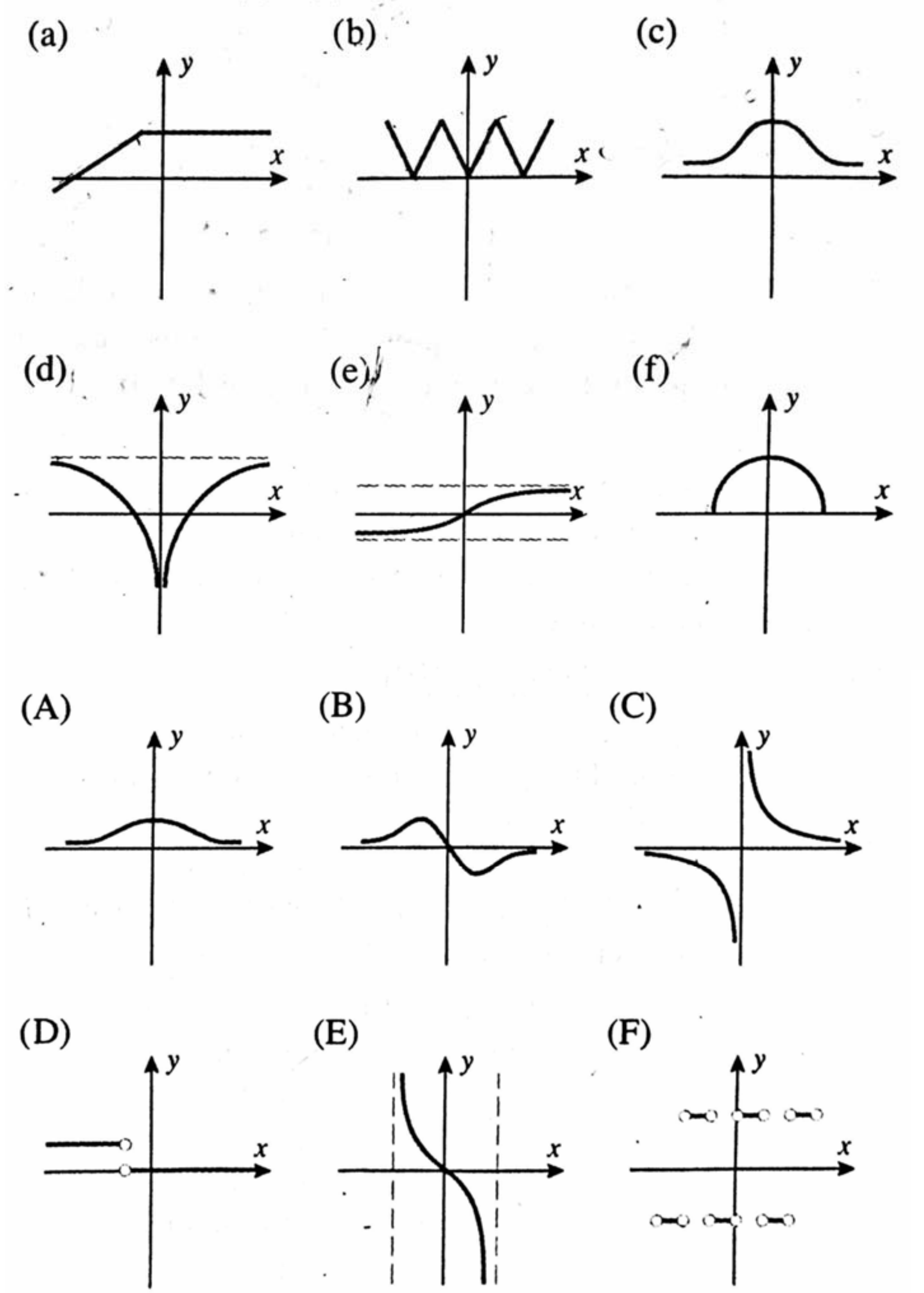
$$f(x) = \begin{cases} \dfrac{\tan kx}{x} & \text{if} \quad x < 0 \\ 3x + 2k^2 & \text{if} \quad x \geq 0 \end{cases}$$

continuous at x = 0.

26. A particle moves on a line away from its initial position so that after t hours it is $s = 3t^2 + t$ miles away from its initial position.

(a) Find the average velocity of the particle over the interval [1, 3].
(b) Find the instantaneous velocity at $t = 1$.
(c) Find the value of *t* on the interval [1, 3] where the instantaneous velocity is equal to the average velocity.

27. Match the graphs of the functions shown in (a) – (f) with the graphs of their derivatives in (A) – (F).



28. Given the following table of values, find the indicated derivatives:

x	$f(x)$	$f'(x)$
2	1	7
8	5	− 3

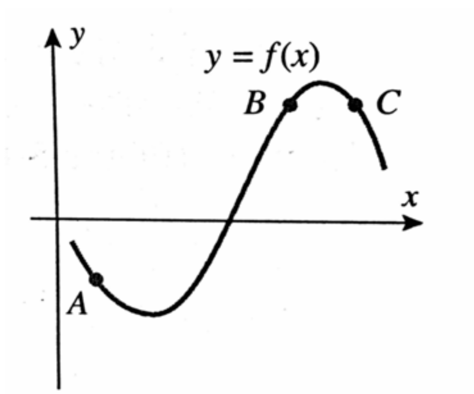
(a) $g'(2)$ where $g(x) = [f(x)]^3$

(b) $h'(2)$ where $h(x) = f(x^3)$

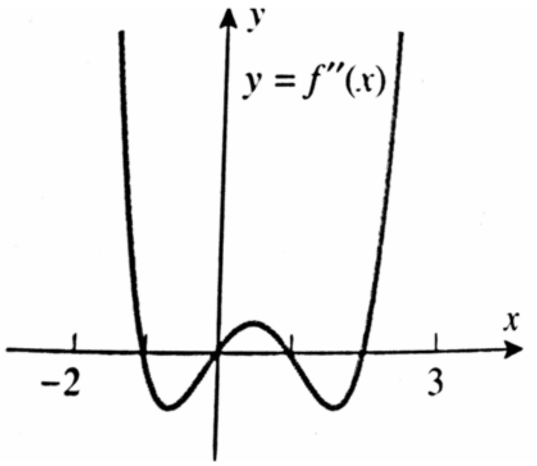
29. Given $f(x) = \tan\left(4x^2\right)$, find $f'(x)$.

30. Given $f(x) = \sin\left(\dfrac{1}{x^2}\right)$, find $f'(x)$.

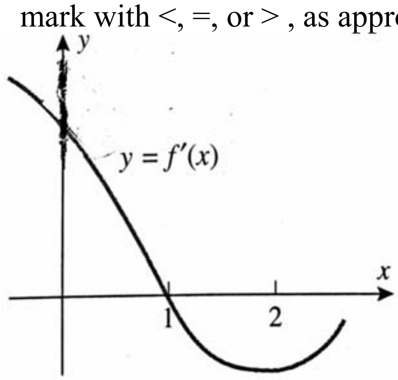
31. Use the graph of the equation $y = f(x)$ in the accompanying figure to find the signs of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at the points A, B, and C.



32. Use the graph of $y = f''(x)$ in the accompanying figure to determine the x-coordinates of all inflection points of f. Explain your reasoning.



33. Use the graph of $y = f'(x)$ in the accompanying figure to replace the question mark with <, =, or > , as appropriate. Explain your reasoning.



(a) $f(0)?\,f(1)$

(b) $f(1)?\,f(2)$

(c) $f'(0)?0$

(d) $f'(1)?0$

(e) $f''(0)?0$

(f) $f''(2)?0$

For #s 34 and 35, find (a) intervals on which f is increasing, (b) intervals on which f is decreasing, (c) intervals on which f is concave up, (d) intervals on which f is concave down, and (e) x-coordinates of all points of inflection on f.

34. $f(x) = 3x^4 - 4x^3$

35. $f(x) = e^{-\frac{x^2}{2}}$

36. Given $y = \sin^{-1}\left(\frac{1}{3}x\right)$, find $\frac{dy}{dx}$.

37. Verify that the hypotheses of the Mean Value Theorem are satisfied on the given interval and find all values of *c* in that interval that satisfy the conclusion of the theorem for $f(x) = \sqrt{x+1}$ on the interval [0, 3].

38. Given $y = \sqrt{\ln x}$, find $y'(e^5)$.

39. Given $y = e^{7x}$, find $y'(\ln 5)$.

40. Given $y = \pi^{\sin x}$, find $y'(\pi)$.

41. Find the absolute maximum and minimum values of *f* on the given closed interval and state where those values occur for $f(x) = \sin x - \cos x$ on the interval $[0, \pi]$.

42. Oil spilled from a ruptured oil tanker spreads in a circle whose area increases at a constant rate of 6 square miles per hour. How fast is the radius of the spill increasing when the area is 9 square miles?

43. The position of a particle moving along the line $y = 2$ is given by $x(t) = 2t^3 - 21t^2 + 60t - 50$ where t is the time in seconds, $t \geq 0$ and x is the position in feet from the point $(0,2)$.

(a) At what time(s) is the particle at rest?
(b) At what time(s) is the particle moving to the right?
(c) At what time(s) is the particle moving to the left?
(d) What is the maximum speed of the particle on the interval $[1,6]$?
(e) On what interval is the velocity of the particle increasing?
(f) What is the total distance travelled by the particle on the interval $[1,6]$?
(g) On what interval(s) is the speed increasing?
(h) On what interval(s) is the speed decreasing?